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$\int$	$\frac{1}{x}$
$2x+1$	$\ln x$
2	$x\ln x - x$
0	

Top Heavy Integrals

A. $\int \frac{x^2+x}{x}\,dx = \int x^{-1}(x^2+x)\,dx = \int x+1\,dx$

$= \frac{1}{2}x^2 + x + C$

$\int \frac{x^2+x}{x} = \int \frac{x^2}{x} + \frac{x}{x} = \int x+1 = \frac{1}{2}x^2 + x + C$

B. $\int \frac{\sqrt{x}+5}{x}\,dx = \int x^{-1}\left(x^{1/2}+5\right)dx$

$= \int x^{-1/2} + 5x^{-1} = \int x^{-1/2} + \frac{5}{x}$

$= 2x^{1/2} + 5\ln|x| + C$

C. $\int \frac{x^3+2x}{\sqrt{x}}\,dx = \int x^{-1/2}\left(x^3+2x\right)dx = \int x^{5/2} + 2x^{1/2}$

$= \frac{2}{7}x^{7/2} + \frac{4}{3}x^{3/2} + C$

What you'll Learn About

- How integrate a fraction when the denominator can be factored and the numerator is not the derivative of the denominator

① Factor denominator

② Build Partial Fractions

A) $\int \frac{x-12}{x^2-4x}dx = \int \frac{x-12}{x(x-4)}dx = \int \frac{3}{x} + \frac{-2}{x-4}$

$= 3\ln|x| - 2\ln|x-4| + C$

$= \ln|x^3| - \ln|(x-4)^2| + C$

$= \ln\left|\frac{x^3}{(x-4)^2}\right| + C$

$\frac{x-12}{x(x-4)} = \frac{A(x-4)}{x(x-4)} + \frac{B(x)}{(x-4)(x)}$ ← $x=0$ $x=4$

$x-12 = A(x-4) + Bx$

$x-12 = Ax - 4A + Bx$

x-terms: $x = Ax + Bx$, $1 = A + B$

Constants: $-12 = -4A$, $3 = A$, $B = -2$

$x=0$: $-12 = -4A$, $3 = A$

$x=4$: $-8 = 4B$, $-2 = B$

B) $\int \frac{16-x}{x^2+3x-10}dx = \int \frac{16-x}{(x+5)(x-2)}dx = \int \frac{-3}{x+5} + \frac{2}{x-2}$

$\frac{16-x}{(x+5)(x-2)} = \frac{A}{(x+5)} + \frac{B}{(x-2)}$

$16-x = A(x-2) + B(x+5)$

$x=-5$: $21 = -7A$, $-3 = A$

$x=2$: $14 = 7B$, $2 = B$

$\frac{A}{x-2} + \frac{B}{x+5}$

$\frac{2}{x-2} - \frac{3}{x+5}$

$= -3\ln|x+5| + 2\ln|x-2|$

$= \ln(x+5)^{-3} + \ln(x-2)^2$

$= \ln(x+5)^{-3}\cdot(x-2)^2$

$= \ln\left|\frac{(x-2)^2}{(x+5)^3}\right| + C$