

Page 31

Top Heavy Integrals

$$\begin{array}{c} \int (x^2+x) dx \\ \hline 2x+1 \\ \hline 2 \\ \hline 0 \end{array}$$

$\left| \frac{1}{x} \right.$   
 $\left. \ln x \right.$   
 $x\ln x - x$

$$A. \int \frac{x^2+x}{x} dx = \int x^{-1}(x^2+x) dx = \int x+1 dx$$

$$= \frac{1}{2}x^2 + x + C$$

$$\int \frac{x^2+x}{x} = \int \frac{x^2}{x} + \frac{x}{x} = \int x+1 = \frac{1}{2}x^2 + x + C$$

$$B. \int \frac{\sqrt{x+5}}{x} dx = \int x^{-1}(x^{1/2}+5) dx$$

$$= \int x^{-1/2} + 5x^{-1} = \int x^{-1/2} + \frac{5}{x}$$

$$= \boxed{2x^{1/2} + 5\ln|x| + C}$$

$$C. \int \frac{x^3+2x}{\sqrt{x}} dx = \int x^{-1/2}(x^3+2x) dx = \int x^{5/2} + 2x^{1/2}$$

$$= \boxed{\frac{2}{7}x^{7/2} + \frac{4}{3}x^{3/2} + C}$$



CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy  
 Chapter 6: Differential Equations      6.5 Partial Fractions

What you'll Learn About

- How integrate a fraction when the denominator can be factored and the numerator is not the derivative of the denominator

① Factor denominator

② Build Partial Fractions

$$\begin{aligned} x=0 & \quad x=4 \quad \leftarrow \frac{x-12}{x(x-4)} = \frac{A(x-4)}{x(x-4)} + \frac{B(x)}{(x-4)(x)} \\ x=0 & \quad \leftarrow x-12 = A(x-4) + BX \\ -12 & = -4A \quad (3=A) \\ x=4 & \quad \leftarrow x-12 = Ax - 4A + BX \\ -8 & = 4B \quad (-2=B) \\ 1 & = A+B \end{aligned}$$

$$\begin{array}{ll} \text{X-terms} & \text{constants} \\ \hline x = Ax + BX & -12 = -4A \\ 1 = A + B & (3 = A) \\ B = -2 & \end{array}$$

$$\frac{A}{x-2} + \frac{B}{x+5}$$

$$\begin{aligned} x=-5 & \quad x=2 \\ 21 & = -7A \quad 14 = 7B \\ (-3 = A) & \quad (2 = B) \end{aligned}$$

$$B) \int \frac{16-x}{x^2+3x-10} dx = \int \frac{16-x}{(x+5)(x-2)} dx = \int \frac{-3}{2x+5} + \frac{2}{x-2}$$

$$\begin{aligned} \frac{16-x}{(x+5)(x-2)} & = \frac{A}{(x+5)} + \frac{B}{(x-2)} \\ 16-x & = A(x-2) + B(x+5) \end{aligned}$$

$$\frac{2}{x-2} - \frac{3}{x+5}$$

$$\begin{aligned} & = 3\ln|x| - 2\ln|x-4| + C \\ & = \ln|x^3| - \ln|(x-4)^2| + C \\ & = \ln \left| \frac{x^3}{(x-4)^2} \right| + C \end{aligned}$$

$$\begin{aligned} & = -3\ln|x+5| + 2\ln|x-2| \\ & = \ln(x+5)^{-3} + \ln(x-2)^2 \\ & = \ln \left( (x+5)^{-3} \cdot (x-2)^2 \right) \\ & = \ln \left| \frac{(x-2)^2}{(x+5)^3} \right| + C \end{aligned}$$